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INFLUENCE OF A MAGNETIC FIELD ON THE PULSATIONAL STABILITY OF STARS

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ABSTRACT

Under certain simplifying assumptions, the influence of a magnetic field on the pulsational stability of stars has been investigated, with a particular application to the problem of the stability of upper–main-sequence stars. It has been found that, if the magnetic field averaged over a spherical shell is either constant at all layers or distributed such that ν, the ratio of magnetic to thermodynamic pressure ($\langle H^2 \rangle / 24\pi P$), is constant at all layers, the critical mass for stability against nuclear-energized pulsations is virtually unaffected by the presence of the field. On the other hand, if the field is strong in the envelope but weak in the core of the star, the critical mass is considerably increased; when ν exceeds about 0.1 in the envelope, stability is attained at all masses.

Subject headings: stars: interiors — stars: magnetic — stars: pulsation

I. INTRODUCTION

Coherent magnetic fields are known to exist on the surfaces of many bright main-sequence stars. How deeply these fields extend is a matter of conjecture, but if they are "fossil" fields, built up from an interstellar seed field during the pre–main-sequence phase of evolution, they could be very intense throughout the body of the star (Cowling 1965). A modifying influence, however, arises from the existence of rotational and convective circulation currents, which are likely to have tangled the field lines in a complex manner, and which may also have expelled from the star most of the exterior field lines or else have pulled them well below the observed surface (Mestel 1965). A complicated interior field geometry may in fact be essential in order to have preserved the field against destructive hydromagnetic instabilities (Tayler 1974). Hence it is of interest to ask what effect a strong, but well-tangled, interior magnetic field has on the pulsational properties of upper-main-sequence stars.

It is already known in the case of nonmagnetic, nonrotating stars that, if the stellar mass exceeds a certain limit, the nuclear reactions occurring in the central regions induce a violent pulsational instability of the whole star, provided that the damping in the outer layers due to local opacity variations is not too severe. This instability develops most strongly in the fundamental radial mode of pulsation. However, it has been found to be readily quenched by fast axial rotation of the star. The purpose of the present paper is to determine how the pulsational stability is affected by an assumed magnetic field.

II. BASIC EQUATIONS

In order to make the problem tractable, a number of simplifying approximations are introduced. First, the magnetic field is assumed to be sufficiently small in scale and sufficiently random in orientation that the net force it exerts can be considered to be approximately radial. Then the problem can be treated as a spherically symmetric one. No limit on the intensity H of the magnetic field need be imposed, but the degree of tangling of the field lines is limited by the requirement that they be not twisted into such small loops that a significant decay of the field would have occurred during an observable portion of the star's lifetime. Since the decay time is, in order of magnitude, given by

$$\tau = 4\pi\sigma l^2 \tag{1}$$

(where l is a characteristic length and the electrical conductivity $\sigma \approx 2 \times 10^{-14} T^{3/2}$ emu [Cowling 1953]), a lower limit to the size of the long-lasting flux loops can be fixed, in units of the stellar radius, at $l/R \approx 10^{-2}$ near the surface and $\sim 10^{-4}$ near the center, if τ is set equal to 10^6 yr, which is appropriate to the massive stars in question here. These loop sizes turn out to be small enough to confirm our assumption that the field could be well tangled and yet persist for a significant length of time. It is also possible to demonstrate that, because of the high electrical conductivity, the gradual decay of the magnetic field generates Joule heat at a rate that is negligible in comparison with the star's radiative output. The rate of heating per unit mass is, in order of magnitude, given by (Cowling 1953)

$$j^2/\sigma\rho = H^2/16\pi^2 l^2 \sigma\rho \,. \tag{2}$$

Taking the magnetic energy density $H^2/8\pi$ to be at most equal to the thermodynamic energy density and using for l the lower limits derived above, we find that $j^2/\sigma\rho$ must be less than $\sim 10^{-1}$ ergs g^{-1} s^{-1} near the stellar surface and less than ~ 10 ergs g^{-1} s^{-1} near the stellar center. These values are insignificant when compared with the mean rate of energy production inside a massive star, $L/M \approx 10^4$ ergs g^{-1} s^{-1}.

Next we turn to a discussion of the equations that will actually be needed for the problem. The general equation of motion for the fluid in the presence of a magnetic field $\boldsymbol{H}$ is, in conventional notation,

$$\rho \frac{dv}{dt} = -\nabla P + \rho g + \frac{1}{4\pi}(\nabla \times \boldsymbol{H}) \times \boldsymbol{H}. \quad (3)$$

Trasco (1970) has shown how the vectorial Lorentz force can be reduced to a mean radial form, by defining

$$f_{\text{mag}}(r) = \frac{1}{4\pi}\int_{-1}^{1}\int_{0}^{2\pi}\left[\frac{1}{4\pi}(\nabla \times \boldsymbol{H}) \times \boldsymbol{H}\right]\cdot \mathbf{1}_r d\mu d\phi, \quad (4)$$

where $\mathbf{1}_r$ is a unit vector in the r-direction and the integral extends over a spherical shell. If, following Trasco, we also define

$$\langle H^2 \rangle = \tfrac{1}{2}\int_{-1}^{1} \boldsymbol{H}\cdot\boldsymbol{H} d\mu \quad (5)$$

and use the assumption that the field is small-scale and random in order to write for the mean radial component $\langle H_r^2 \rangle = \frac{1}{3}\langle H^2 \rangle$, then equation (4) reduces to the simple form

$$f_{\text{mag}}(r) = -\frac{\partial}{\partial r}\left(\frac{1}{3}\frac{\langle H^2 \rangle}{8\pi}\right) \quad (6)$$

for the axially symmetric case. Hence the reduced equation of motion of the fluid becomes

$$\rho \frac{d^2 r}{dt^2} = -\frac{\partial}{\partial r}\left(P + \frac{\langle H^2 \rangle}{24\pi}\right) - \frac{GM(r)\rho}{r^2}. \quad (7)$$

It is apparent from this equation that the total pressure may be regarded as the sum of a thermodynamic pressure P and a magnetic pressure $\langle H^2 \rangle/24\pi$.

If we now assume for simplicity that $\langle H^2 \rangle/24\pi \propto P^a$, where a is a constant, and if we define

$$\nu = \langle H^2 \rangle / 24\pi P, \quad (8)$$

then the hydrostatic version of equation (7) can be written

$$\frac{dP}{dr} = -\left(\frac{1}{1 + a\nu}\right)\frac{GM(r)\rho}{r^2}. \quad (9)$$

The other basic equations of static stellar structure remain the same, to our degree of approximation, as in the absence of a magnetic field. Therefore, the only effect of the magnetic field is to weaken the gravitational force by an amount $(1 + a\nu)^{-1}$.

Radial perturbations of the equilibrium models will next be computed in the linearized adiabatic approximation by using the equations for the nonmagnetic case given by Schwarzschild and Härm (1959) but suitably modified to allow for the presence of (a) permanent magnetic fields and (b) opacity sources other than pure electron scattering.

The rate of change of the magnetic field following the motion of a mass element of the fluid is given by

$$\frac{d\boldsymbol{H}}{dt} = \frac{\partial \boldsymbol{H}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{H}. \quad (10)$$

For highly conducting material like the ionized gases inside stars, the equation of electromagnetic induction can be written

$$\frac{\partial \boldsymbol{H}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{H}). \quad (11)$$

Assuming small periodic oscillations about the equilibrium state, we may set, in the usual way, $\boldsymbol{r} = \boldsymbol{r}_0 + \delta \boldsymbol{r} e^{i\sigma t}$, etc., where σ is here an angular oscillation frequency. With the help of the relation $\nabla\cdot\boldsymbol{H} = 0$, equation (10) may be brought into the form, correct to first order in small quantities,

$$\delta\boldsymbol{H} = (\boldsymbol{H}_0\cdot\nabla)\delta\boldsymbol{r} - \boldsymbol{H}_0(\nabla\cdot\delta\boldsymbol{r}). \quad (12)$$

It is required to compute

$$\delta\langle H^2 \rangle = \int_{-1}^{1} \boldsymbol{H}_0\cdot\delta\boldsymbol{H} d\mu. \quad (13)$$

Supposing the oscillations to be purely spherically symmetric, and using the relations $\delta\rho/\rho_0 = -\nabla\cdot\delta\boldsymbol{r}$ and $\langle H_r^2 \rangle = \frac{1}{3}\langle H^2 \rangle$, we obtain after some algebra the simple result

$$\frac{\delta\langle H^2 \rangle}{\langle H^2 \rangle} = \frac{4}{3}\frac{\delta\rho}{\rho}. \quad (14)$$

Here the zero subscripts have been dropped for convenience. Equation (14), being based on equation (11), expresses the fact that the magnetic lines of force are carried along with the moving fluid, i.e., that the magnetic flux is locally conserved.

Next, by perturbing equation (7) and by assuming that the stellar gas is completely ionized and is oscillating adiabatically, we find

$$x\frac{d}{dx}\left(\frac{\delta P}{P}\right) = \left(\frac{V}{1 + 4\nu/3\Gamma_1}\right) \times \left[\left(1 + \frac{4\nu}{3\Gamma_1}a - \frac{4\nu}{3\Gamma_1}b\right)\frac{\delta P}{P} + (1 + a\nu)\left(4 + \frac{\omega^2 x^3}{q}\right)\frac{\delta r}{r}\right], \quad (15)$$

where

$$b = \frac{(1-\beta)(7\Gamma_1 - 8 - 2\beta)}{\Gamma_1(8 - 7\beta)}\left(1 - \frac{4}{n+1}\right) \quad (16)$$

and the usual nondimensional Schwarzschild (1958) variables have been adopted. The other parameters include the square of the nondimensional pulsational eigenfrequency $\omega^2 = \sigma^2 R^3/GM$, where $\sigma = 2\pi/\text{period}$; the generalized adiabatic exponent Γ_1; the local effective polytropic index n; and the homology invariant

$V = -d\ln P/d\ln r$. For an arbitrary opacity law, the radiative luminosity perturbation is given by

$$\frac{\delta L(r)}{L(r)} = 4\frac{\delta r}{r} - \alpha\frac{\delta\rho}{\rho} + (4+\eta)\frac{\delta T}{T} + T\frac{dr}{dT}\frac{d}{dr}\left(\frac{\delta T}{T}\right), \tag{17}$$

where the thermodynamic derivatives of opacity are $\alpha = \partial\ln\kappa/\partial\ln\rho$ and $\eta = -\partial\ln\kappa/\partial\ln T$. The rest of the stellar perturbation equations are the same as in Schwarzschild and Härm's (1959) paper, except for a correction of some very minor errors in their equations and for a suppression of the terms referring to atmospheric running waves. The stability integral, which indicates whether the star is stable or unstable, is unchanged in the presence of a magnetic field when the quasi-adiabatic form given by Schwarzschild and Härm (their eq. [8]) is used.

III. NUMERICAL RESULTS

To afford a comparison with earlier work on nonmagnetic stellar models, the basic physical formulae adopted by Stothers and Simon (1970) have been used here, in conjunction with two different opacity representations: (1) Thomson scattering by free electrons and (2) Cox-Stewart opacities in the form given in the paper just cited. For the magnetic field intensity, three representative spatial distributions have been chosen: (1) a uniform mean field intensity (i.e., $\langle H^2\rangle$ = constant); (2) a nonuniform mean field intensity, with ν = constant everywhere; and (3) the same as case (2), but with no magnetic field allowed in the convective core. For simplicity, it has been assumed that the adopted spatial distribution in the envelope holds formally up to the top of the atmosphere; the substitution of more realistic surface boundary conditions has been found, by supplementary calculations, to affect the solutions very little (see Appendix).

A uniform mean magnetic field is a simple example of a force-free field. It has no effect on the equilibrium structure of the star (except at the surface). On the other hand, the example of a nonuniform field with ν = constant everywhere strongly concentrates the field toward the center of the star and provides an effective force that counteracts gravity. Such a field would result from the contraction of a primordial gas cloud that was initially permeated by a uniform magnetic field if the magnetic lines of force remained frozen into the material during contraction. It is easily seen that this consequence follows from equation (14) since the magnetic field strength of a mass shell of the cloud would increase in time as $\langle H^2\rangle \propto \rho^{4/3}$, while the final structure of the star must be roughly polytropic with $P \propto \rho^{4/3}$. In this idealized case, a straightforward similarity analysis of the basic equations of stellar structure leads to a rough prediction of how the surface properties of the star ought to change with the strength of the magnetic field, viz., $R \sim$ constant; $L \sim (1+\nu)^{-4}$ for a high stellar mass; and $L \sim (1+\nu)^{-1}$ for an "infinite" stellar mass. For greater refinement in these predictions, the detailed models for 54 $M_\odot$ in Table 1 should be consulted. The relatively strong dependence of luminosity on ν can be used, in conjunction with the observed mass-luminosity relation, to set a limit of $\nu < 0.2$ for upper–main-sequence stars.

The foregoing picture, however, is considerably oversimplified. Turbulent convection, particularly in the large core of the star, would probably severely twist the magnetic field lines within the superadiabatic layers, thus causing them to decay rapidly, or would possibly expel them into the radiative part of the envelope, where they could then diffuse upward, or even dissipate, as a result of hydromagnetic instabilities. Of some physical interest, then, is the case where a strong magnetic field pervades the envelope but where the core is largely nonmagnetic. This is, in fact, our third case listed above. In order to derive the convective core boundary, we have demanded equality of the radiative and adiabatic temperature gradients on the interior side of the boundary, where the convective velocities in the core should vanish, and we have

TABLE 1

CRITICAL MASSES FOR THE PULSATIONAL STABILITY OF MASSIVE MAGNETIC STARS WITH $(X, Z) = (0.70, 0.03)$*

Opacity	Envelope Mean Magnetic Field	Core Mean Magnetic Field	ν_R	$M/M_\odot$	$\log(L/L_\odot)$	$\log T_e$	$\log(R/R_\odot)$	β_c	$\rho_c/\langle\rho\rangle$	ω^2
Thomson.......	Zero	Zero	0	54	5.67	4.70	0.96	0.65	21	3.0
	Uniform	Uniform	0.10	54	5.67	4.70	0.96	0.65	21	3.0
			1.00	57	5.71	4.70	0.97	0.64	21	2.9
	Nonuniform	Nonuniform	0.10	54	5.57	4.68	0.95	0.68	20	2.9
	Nonuniform	Zero	0.08	95	6.09	4.72	1.14	0.54	30	2.8
			0.10	115	6.23	4.72	1.20	0.50	33	2.7
			0.14	~300	6.84	4.73	1.48	0.34	57	2.5
Cox-Stewart....	Zero	Zero	0	106	6.15	4.72	1.16	0.51	29	2.8
	Uniform	Uniform	0.10	107	6.15	4.72	1.16	0.51	29	2.8
			1.00	112	6.18	4.72	1.17	0.50	29	2.7
	Nonuniform	Nonuniform	0.10	94	5.97	4.70	1.12	0.56	28	2.9
	Nonuniform	Zero	0.08	~500	7.09	4.73	1.61	0.27	66	2.4

* ν = constant in regions with a nonuniform mean magnetic field.

assumed that, on the exterior side, the presence of the magnetic field quenches any local convective instability. How the Schwarzschild criterion for convective instability is to be modified to take account of a magnetic field is unclear (Tayler 1971), but in any case small modifications of the convective core boundary in our stellar models are found to change the resulting structures rather little.

Alternatively, if a strong magnetic field exists in the core, it might completely suppress the convection there. At present, it is not known whether, for this to happen, the magnetic pressure must be comparable with gas pressure or only with turbulence pressure (which is, in the core, much weaker than gas pressure). Current evidence seems to favor the existence of large convective cores in upper–main-sequence stars (Stothers and Chin 1973). Therefore, any magnetic field in the core probably generates pressures that are, at least, not enormously stronger than turbulence pressure. These remarks serve to elucidate our particular choices for the magnetic field distributions mentioned above.

Vibrational stability of the stellar models in the fundamental radial mode of pulsation has been tested for various assumed strengths of the magnetic field. Results for the "critical" masses dividing stable from unstable models are given in Table 1, where the nonmagnetic models listed for the two adopted opacity representations are those derived originally by Schwarzschild and Härm (1959) and by Stothers and Simon (1970). In this table, ν_R refers to the magnetic field in the stellar photosphere (a value of 1 corresponding to $H \approx 2000$ gauss).

It is, at first sight, surprising how slight a dependence on magnetic field strength is shown by the critical masses in the cases where $\langle H^2 \rangle$ is either constant or distributed according to the law ν = constant. In fact, the actual dependence must be even milder than is indicated by the quasi-adiabatic calculations if nonadiabatic effects near the surface are taken into account. These effects become significant for large values of ν. If they are estimated in the manner suggested by Stothers and Simon (1970), then even for a surface field as intense as 34,000 gauss—the largest ever observed in a main-sequence star—the assumption of a uniform mean field yields virtually the same critical mass as the case of no field at all. The physical explanation of these numerical results will be given in § IV.

In the last case, where the envelope of the star is assumed to possess a strong magnetic field but the core little or no field, the reduction of gravity in the outer layers increases the central condensation of the star and hence lowers the pulsation amplitudes near the center, where the nuclear driving sources lie. In consequence, the critical mass shifts to a large enough value that radiation pressure—a destabilizing influence—overcomes the damping effect of the increased central condensation. It turns out that, at some further mass value beyond the critical mass, the increased central condensation eventually overcomes radiation pressure and pulsational stability is recovered. A numerical example will best illustrate these results. Consider the case of Thomson scattering as the opacity. If $\nu = 0$, then the critical mass is found to be 54 $M_\odot$, the models with higher masses all being unstable. When ν is increased to 0.10, the critical mass shifts upward to 115 $M_\odot$, but the models heavier than $\sim 1800\ M_\odot$ are found to be stable. For a sufficiently large value of ν, pulsational stability is attained at all masses. In the present example, this limit is reached when $\nu = 0.14$, in which case stability is minimal at $\sim 300\ M_\odot$. If the Cox-Stewart opacities are adopted, the corresponding values of ν and of stellar mass are 0.08 and $\sim 500\ M_\odot$.

IV. ANALYTIC RESULTS

a) *Uniform Mean Magnetic Field*

For stellar models endowed with a general magnetic field, Chandrasekhar and Limber (1954) have computed an approximate expression for the square of the pulsational eigenfrequency:

$$\sigma^2 = (3\langle\Gamma_1\rangle - 4)(-W - E_{\rm mag})/I\,, \tag{18}$$

where I is the total moment of inertia of the star with respect to the center, W is the total gravitational potential energy, and $E_{\rm mag}$ is the total magnetic energy. If V is the volume, then $E_{\rm mag} = \int (H^2/8\pi)dV$. A conveniently reduced expression for σ^2 has already been derived for the nonmagnetic case (Stothers 1974). Including the magnetic field, we here obtain[1]

$$\omega^2 = \frac{3\langle\Gamma_1\rangle - 4}{2n - 1}\frac{\rho_c}{\langle\rho\rangle} - (3\langle\Gamma_1\rangle - 4)\frac{E_{\rm mag}}{0.1GM^2/R}\,, \tag{19}$$

where the approximate relation $I = 0.1R^2M$ has been introduced. For the case of a *uniform* mean field of any reasonable intensity, the total magnetic energy will be very small compared with the total gravitational potential energy since the magnetic pressure is significant only in the outermost layers. Therefore, the second term in equation (19) can be dropped.

For a nonmagnetic star, the equilibrium structure can be approximated satisfactorily by Eddington's (1926) standard model. This model is a polytrope of index $n = 3$, with $\rho_c/\langle\rho\rangle = 54$ and β = constant. Associated with it is Eddington's quartic equation,

$$(M/M_\odot)\mu^2 = 18(1 - \beta)^{1/2}\beta^{-2}\,, \tag{20}$$

which provides the link between the ratio of gas pressure to total thermodynamic pressure β and the stellar mass M if the mean molecular weight μ is specified. Thus $\langle\Gamma_1\rangle$, being a unique function of β, is immediately known. Since, in a magnetic star, a uniform mean field exerts no net radial force, equation (20) remains *unchanged* when such a field is present.

Equations (19) and (20), together with the determining condition for the "critical" mass $\omega^2 \approx 3$ (§ III; Simon and Stothers 1969), define the problem completely. It is evident from a simple inspection of

[1] A wider range of applicability results if one replaces the factor $2n - 1$ (valid for $1.5 \leq n \leq 3$) by the factor $1 + n^2/2$ (valid for $0 \leq n \leq 3$).

this set of equations that the critical mass will be essentially independent of the adopted strength of the magnetic field.

b) Nonuniform Mean Magnetic Field

Consider the case where the ratio of magnetic pressure to thermodynamic pressure is equal to a constant ν. Then $E_{\text{mag}} = \nu \int 3PdV$. The virial theorem, modified to include a general magnetic field, can be written (Chandrasekhar and Fermi 1953)

$$\int 3PdV + W + E_{\text{mag}} = 0 \,. \tag{21}$$

It follows in the present case that $E_{\text{mag}} = -\nu(1+\nu)^{-1}W$. In place of equation (19), we therefore have

$$\omega^2 = \frac{3\langle\Gamma_1\rangle - 4}{(1+\nu)(2n-1)} \frac{\rho_c}{\langle\rho\rangle} \,. \tag{22}$$

Under the present assumption of constant ν, the only change required in the equilibrium equations of stellar structure is the replacement of G by another constant $(1+\nu)^{-1}G$. Consequently, the central condensation, as measured by $\rho_c/\langle\rho\rangle$, remains unaffected. On the other hand, Eddington's quartic equation now becomes

$$(M/M_\odot)\mu^2 = 18(1-\beta)^{1/2}\beta^{-2}(1+\nu)^{3/2} \,. \tag{23}$$

Thus the two direct effects of introducing a magnetic field with constant ν are to increase $\langle\Gamma_1\rangle$ by lowering the relative radiation pressure (i.e., $1-\beta$) and to "screen" the gravitational force by an amount $(1+\nu)^{-1}$. These two factors, the one stabilizing and the other destabilizing, compensate each other almost exactly and so leave ω^2, and hence the critical mass, nearly constant.

One further case is of physical interest here. If ν is very small in the stellar core but significant in the stellar envelope, the critical mass is no longer a constant. This conclusion follows from the fact that the two variables $P + \langle H^2\rangle/24\pi$ and T must both be continuous at all layers in the star, and therefore the density ρ, under the present hypothesis, must experience a sudden decrease on the exterior side of the core boundary. (The effect is identical to that produced by a discontinuity in the mean molecular weight at the core boundary of an evolved star.) The induced growth of the central condensation, as measured by $\rho_c/\langle\rho\rangle$, raises ω^2, and hence also raises the critical mass. Conversely, if ν is significant only in the stellar core, the critical mass is lowered.

V. CONCLUSION

If the magnetic field inside a homogeneous main-sequence star can be assumed to be approximately axisymmetric and subject to the local condition $\langle H_r^2\rangle = \frac{1}{3}\langle H^2\rangle$, then such a field is found to have the following consequences, depending on the field's spatial distribution: First, a uniform mean field produces virtually no change in the equilibrium and pulsational properties of the star. Second, a mean field for which the ratio of magnetic to thermodynamic pressure $(\langle H^2\rangle/24\pi P)$ is large and constant everywhere in the star leads to a greatly reduced luminosity of the star and to a mildly reduced effective temperature, but to no significant change in the pulsational properties. Third, a mean field that is strong in the stellar envelope and weak in the stellar core lowers the luminosity relatively little (for a large core mass) but mildly reduces the effective temperature, and tends to stabilize the star pulsationally. In fact, if the ratio of magnetic to thermodynamic pressure in the envelope is greater than or equal to 0.1, the star is found to be stable against nuclear-energized pulsations regardless of how high its mass is.

Of greatest astrophysical interest is the third case. It suggests that, if magnetic fields are indeed prevalent and strong in the envelopes of upper–main-sequence stars, the search for nuclear-destabilized stars of high mass will turn out to be fruitless. This conclusion is only strengthened if the stars have a moderate amount of rotational angular momentum as well.

It is a pleasure to thank Dr. C.-w. Chin for independently verifying equation (6).

APPENDIX

SURFACE BOUNDARY CONDITIONS

The geometry of the surface magnetic field may be rather simple (e.g., dipolar), but for heuristic purposes the angular average of the Lorentz force given by equation (6) will be adopted here. At some large distance above the photosphere of the star, the total pressure, including the magnetic pressure, must essentially vanish. Let us assume that, at all layers above the photosphere, the magnetic pressure $\langle H^2\rangle/24\pi$ falls off like r^{-c}, where c is a positive constant. Since the condition $dP/dr < 0$ must hold everywhere, we require, at large r, $c > 1$ and, at the photosphere,

$$c < \frac{GM}{R}\left(\frac{\langle H^2\rangle}{24\pi\rho}\right)_R^{-1} . \tag{A1}$$

By defining $\nu = \langle H^2\rangle/24\pi P$ and by integrating the equation of hydrostatic equilibrium from the top of the atmosphere down to the photosphere, we find that the inequality (A1) can be expressed approximately as $c < \nu_R^{-1}(R/\Delta r)$, where Δr is the effective thickness of the atmosphere. Since $R/\Delta r$ is normally much greater than unity, the formal upper limit on c is very large unless the magnetic field strength greatly exceeds the largest value that has ever been observed in a main-sequence star.

With the present adoption of an r^{-c} law for the magnetic pressure, the surface boundary condition for the mechanical pulsation equation becomes

$$\left(\frac{\delta P}{P}\right)_R = -\frac{(4+\omega^2-2\gamma)}{(1-\gamma)(1-4\nu b/3\Gamma_1)}\left(\frac{\delta r}{r}\right)_R, \tag{A2}$$

where

$$\gamma = c\left(\frac{GM}{R}\right)^{-1}\left(\frac{\langle H^2\rangle}{24\pi\rho}\right)_R. \tag{A3}$$

If $1 < c < c_{\text{upper}}$, then $0 < \gamma < 1$. For reasonable values of γ (i.e., values close to zero), equation (A2) differs but little from our formally adopted surface boundary condition based on imposing regularity on equation (15):

$$\left(\frac{\delta P}{P}\right)_R = -\frac{(1+a\nu)(4+\omega^2)}{1-4\nu(b-a)/3\Gamma_1}\left(\frac{\delta r}{r}\right)_R. \tag{A4}$$

As for the heat flow condition, regularity of equation (17) requires

$$\left[\frac{\delta L(r)}{L(r)}\right]_R = \left[4\frac{\delta r}{r} - \alpha\frac{\delta\rho}{\rho} + (4+\eta)\frac{\delta T}{T}\right]_R. \tag{A5}$$

This implies that the radiative diffusion approximation holds throughout the atmosphere. We have formally adopted this approximate surface boundary condition.

The making of various changes in the equilibrium and perturbed model atmospheres is found to have little perceptible influence on the deeper layers of the stellar models because the inner structure of a radiative envelope is rather insensitive to the surface boundary conditions.

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